

MTH241 Fall 2024: Exam 03

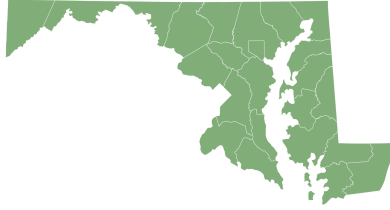
Instructor: Dr. Ling Liang

Name:

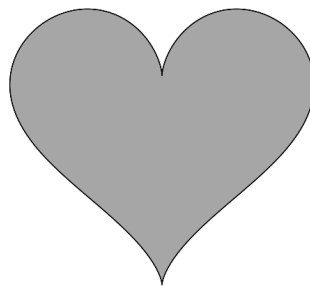
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Closed book, no calculator, show your work clearly. Read the problems carefully.

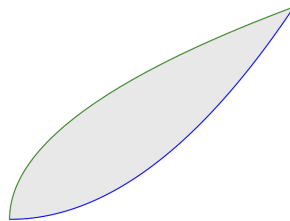
1. (15pt) Determine whether the following regions are vertically simple, or horizontally simple, or both. Check whatever is correct.



Horizontally Simple	Vertically Simple	Simple
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Horizontally Simple	Vertically Simple	Simple
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Horizontally Simple	Vertically Simple	Simple
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2. (20pt) Let R be the region in the plane between $x^2 + y^2 = 1$ and $y = 1 - x$, and in the first quadrant. Sketch the region R in the xy -plane and write down the iterated double integral for $\iint_R xy dA$, treating R as a vertically simple region. Evaluate the integral.

3. (15pt) Let D be the solid region inside the sphere $x^2 + y^2 + z^2 = 4$ and above the cone $z = \sqrt{2x^2 + 2y^2}$. Describe D using spherical coordinate, i.e., find $\alpha, \beta, h_1, h_2, F_1, F_2$ such that D is as

$$\alpha \leq \theta \leq \beta, \quad h_1(\theta) \leq \phi \leq h_2(\theta), \quad F_1(\phi, \theta) \leq \rho \leq F_2(\phi, \theta).$$

Moreover, set up the iterated integral for $\iiint_D x^2 dV$ (Don't evaluate the integral).

4. (10pt) Perform a change of variable to evaluate the integral $\iint_R 1dA$ where R is the region inside $\frac{x^2}{9} + \frac{y^2}{16} = 1$.

5. (15pt) Let the smooth curve C be parameterized by $\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j} + t\vec{k}$ for $t \in [0, 2\pi]$. Evaluate the line integral

$$\int_C \frac{z^2}{x^2 + y^2} ds.$$

6. Let $\vec{F}(x, y) = -y\vec{i} + x\vec{j} + 0\vec{k}$ be a vector field and C be the circle in the xy -plane centered at $(0, 0)$ with radius 1, oriented counterclockwise.

(a) (10pt) Determine whether there exists a function $f(x, y)$ such that $\text{grad } f = \vec{F}$.

(b) (15pt) Evaluate the following integral $\int_C \vec{F} \cdot d\vec{r}$.

Extra page: